values of $P_n(0)$ and $Q_n(0)$ are not given. The tables were calculated on an IBM 7090 computer, and it is stated that the accuracy of the values is about ± 2 in the last significant figure.

The limit $n = 27$ is considerably higher than for previous tables of $P_n(x)$ and especially $Q_n(x)$, though $P_n(\cos\theta)$ has been tabulated for still higher values of n.

Most of the values of $P_n(x)$ up to $n = 16$ may be checked against a 6D table of Tallqvist [1]; this is insufficient near zeros, where Paran & Kagle keep 6S. Other checking tables also exist. A few random comparisons suggest good accuracy in the present table.

In 1945 the Admiralty Computing Service issued a useful little 5D table of $Q_n(x)$, $n = 0(1)7$, $x = 0(0.01)1$, mainly copied from Vandrey with corrections. The reviewer has compared all 792 common values with Paran & Kagle, and a few slight corrections to the ACS table are given in the appropriate section of this issue (p. 335). The table of Paran & Kagle has most often one extra figure, so that the comparison checks it only very partially indeed, but here again one has the impression of good accuracy in the Paran & Kagle values. If these values contain any errors as large as two final units, perhaps they occur for the higher values of n.

This is a "working table" rather than a definitive one, with properly rounded values, but it is important enough to make one glad that it has been classified as for unlimited circulation.

A. F.

1. H. Tallqvist, *Sechsstellige Tafeln der 16 ersten Kugelfunktionen $P_n(x)$*, *Acta. Soc. Sci. Fenn.*, Nova Ser. A, Tom II, No. 4, 43 p., Helsingfors, 1937.

51[L, M].—Yudell L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Company, New York, 1962, xv + 419 p., 23 cm. Price $12.50.

This book professes to deal with definite and indefinite integrals involving Bessel functions (and related functions) and purports to provide the applied mathematician with the basic information relating to such integrals; but, in fact, it does more than it promises. In addition to information relating to Bessel functions, it gives much useful information about other special functions, and the reader learns a great deal about the evaluation, convergent expansion, and asymptotic expansion of integrals involving functions of the hypergeometric type. Within the field of integrals involving Bessel functions, special emphasis is placed on indefinite integrals, since these are somewhat scantily treated in other well-known and easily accessible works of reference where definite integrals are more adequately covered.

Chapter I is preparatory and contains information and useful collections of formulas regarding the gamma function, generalized hypergeometric series, and Bessel functions; in the latter case including polynomial approximations useful for the numerical computation of these functions. A brief list of tables of Bessel functions is appended.

Chapter II is devoted to the integral

$$Wi_{\mu,\nu}(z) = \int_0^z t^\mu W_\nu(t)\,dt \tag{1}$$

in which W is J, Y, $H^{(1,2)}$, I, or K. Since the organization of this chapter is typical of the organization of several other chapters, it is worth considering it in some detail. First the connections between the various Bessel functions, the differential equation satisfied by them, and their power series expansions are used to obtain the corresponding formulas for Ji, Yi, etc. Special cases (ν a non-negative integer) are also considered, and the expansions of

$$\int_z^\infty t^\mu W_\nu(t)\, dt$$

for small z are given in cases when the integrand is not integrable at $t = 0$. Next follow expansions in series of Bessel functions, and asymptotic expansions for large z. These lead to the values of related infinite integrals. Approximations of Bessel functions in terms of trigonometric functions are used to obtain similar approximations for Wi; and polynomial approximations to some Wi, with tabulated values of the numerical coefficients, and error bounds for stated intervals, are also given. The chapter concludes with a list of available numerical tables of Ji, Yi, Ii, Ki. Detailed derivations are not given, but usually there is a reference or enough information to enable a competent analyst to verify the results.

In Chapter III Lommel's functions $S_{\mu,\nu}(z)$ and $s_{\mu,\nu}(z)$ are introduced. For these, and for the Struve functions and Anger-Weber functions, a detailed list of formulas is given. These functions are used to provide alternative expressions for some Wi and related integrals. Somewhat surprisingly, there is a collection of formulas facilitating the expansion of polynomials in a Fourier-Bessel series.

Chapter IV concerns integrals of the form

$$\int^z e^{\epsilon t} t^\mu W_\nu(t)\, dt, \tag{2}$$

where $\epsilon = \pm i$ if W is a Bessel function, and $\epsilon = \pm 1$ if W is a modified Bessel function; the functions corresponding, in this case, to Lommel's functions are also introduced. The organization is similar to that of Chapters II and III.

Chapter V is very brief; it gives five reduction formulas for the integral

$$\int^z e^{-pt} t^\mu W_\nu(\lambda t)\, dt, \tag{3}$$

and then the explicit values of some two dozen special integrals of this form. The latter are also special cases of the integrals of the earlier chapters.

Chapter VI treats Airy functions and their integrals, and Chapter VII treats the incomplete gamma functions, their special cases (error functions, exponential integrals, etc.), and their integrals, in a similar fashion.

Repeated integrals of fractional order, both Riemann-Liouville and Weyl integrals, of Bessel functions form the subject of Chapter VIII. In Chapter IX we find integrals of the form (1), (2), or (3) in which, however, W_ν is a Struve function (rather than a Bessel function). Chapter X is devoted to the integrals

$$\int_0^z e^{it} J_0(\lambda t)\, dt, \qquad \int_0^z e^{it} Y_0(\lambda t)\, dt$$

("Schwarz functions") and to their generalization

$$\int_0^1 e^{i\omega t}(1 - t)^\delta t^\mu J_\nu(\beta t)\, dt.$$

Chapter XI lists a great many indefinite integrals whose integrands contain a product of two Bessel functions, or a product of a Bessel function and a Struve function. The integrals are evaluated by utilizing the differential equations satisfied by these functions. There is also a brief section on an integral over a product of three Bessel functions.

The last chapter on indefinite integrals, Chapter XII, contains a miscellany of integrals that do not fit into the classification of earlier chapters. Some samples are:

$$e^{-y}\int_0^x e^{-t}I_0(2(yt)^{1/2})\,dt; \qquad \int^z e^{-\alpha}(\alpha t^2+\beta)^{-2}t^{\nu}J_{\nu-1}(S)\,dt,$$

where $2\alpha(\alpha t^2+\beta) = \gamma(t^2-1)$, $S(\alpha t^2+\beta) = \gamma t$, and α, β, γ are independent of t; and similar and related integrals.

Many definite integrals can be obtained from the indefinite integrals of the earlier chapters; others, which cannot be so obtained are collected in Chapter XIII. In view of existing collections, the author emphasizes the numerical and analytical results obtained since about 1945 and 1950, respectively, but the more important earlier results are also included. Some of the groups of definite integrals listed here are: integrals expressing orthogonal properties (in Fourier-Bessel and Neumann series); convolution integrals (including Sonine's and related integrals); Lommel's functions of two variables; Hankel's, Weber's, Weber-Schafheitlin's, Sonine-Gegenbauer's, and related infinite integrals; infinite integrals involving products of Bessel functions; integrals with respect to the order. In many of these cases references to numerical tables are given in addition to analytical results. There is also a brief section on dual and triple integral equations.

Chapter XIV (38 p.) contains a useful collection of numerical tables of Bessel functions (10 tables) and their integrals (2 tables); these are extracted from published works.

A bibliography of 18 pages, an index of notation, author index, and subject index complete the volume.

The volume is reproduced by photo offset from typed copy. Both the typing and the reproduction are excellent.

The value of such a compilation depends essentially on how practical the grouping of integrals will prove in actual use, how easy it is (for a non-expert) to find a given integral, and how well the author succeeded in keeping down the number of (inevitable) misprints. Meanwhile, the first impression is decidedly favourable, and there is every prospect of the book becoming a valuable work of reference.

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52[L, M].—H. C. Spicer, *Tables of the Inverse Probability Integral*

$$P = \frac{2}{\sqrt{\pi}}\int_0^\beta e^{-\beta^2}\,d\beta,$$

U. S. Geological Survey, Washington 25, D. C. Deposited in the UMT File.

This manuscript is in the form of original computation sheets. It contains inverse